AP Calculus AB — Activity 1209 - Riemann Integrals — Name:

Dr. Paul L. Bailey — Monday, December 9, 2024

The Riemann integral produces the area between the graph of a function and the x-axis. We now give the formal definition.

As the definition is given in class, fill in the blanks.

Let $a, b \in \mathbb{R}$ with $a < b$.

A *partition* of the closed interval $[a, b]$ is a finite set of the form

with the property that

Let $P = \{x_0, x_1, x_2, \dots, x_n\}$ be a partition of $[a, b]$. We view P as indicating a way of breaking the interval $[a, b]$ into n subintervals. The width of the i subinterval is $\Delta x_i = x_i - x_{i-1}$, for $i = 1, \dots, n$.

The *norm* of the partition P is

A *choice set* for P is a finite set of the form

such that $c_i \in [x_{i-1}, x_i]$, for $i = 1, \dots, n$. Note that this implies

Let $f : [a, b] \to \mathbb{R}$. The *Riemann sum* associated to a partition P and a choice set C for P is

We say that f is *Riemann integrable with integral I* if there exists a real number $I \in \mathbb{R}$ such that, for every positive real number $\epsilon > 0$, there exists a real number $\delta > 0$ such that for every partition P and choice set C of P,

If f is Riemann integrable with integral I, we write

This is read, "the integral from a to b of $f(x)\,dx$ equals I".